Towards a Wind Turbine Wake Reduced-Order Model

Nicholas Hamilton[1,2], Bianca Viggiano[1], Marc Calaf[3], Murat Tutkun[4,5] and Raúl Bayoán Cal[1]

[1]Mechanical and Materials Engineering, Portland State University, Oregon, USA

[2]National Renewable Energy Laboratory, Colorado, USA

[3]Mechanical Engineering, University of Utah, Utah, USA

[4]Department of Process and Fluid Flow Technology, IFE, Kjeller, Norway

[5]Department of Mathematics, University of Oslo, Oslo, Norway

Abstract

A computationally efficient reduced-order model for a wind turbine wake (wakeROM) defined through turbulent velocity fields from large-eddy simulation data. Basis functions of the proper orthogonal decomposition (POD), represent the spatially coherent turbulence structures in the wind turbine wake. A reduced-order model of the wind turbine wake is defined by inter-relating dynamic mode coefficients through a series of polynomial parameters; the resulting system of ordinary differential equations models the dynamics of the wind turbine wake using only large-scale turbulence.

The wakeROM is periodically reinitialized by relating the incoming turbulent velocity to the POD mode coefficients.

Keywords: *POD, reduced-order model, wake dynamics, control*

Introduction

Wind turbine wakes combine effects from the atmospheric boundary layer interacting with large rotating structures, are subject to wake-to-wake interaction within a wind plant, are asymmetrical and reflect the specific nature of incoming inflow. Despite the known complexity of wind turbine wakes, design and control of wind plants rely on models that are not capable of describing the flow physics and lead to wind plant underperformance. In the current work, turbulent dynamics are depicted through large eddy simulations (LES), isolated with the snapshot POD, and used to formulate a reduced-order model. The resulting dynamical system is subject to instabilities endogenous to wake turbulence and introduced by numerical solution of the system. Incoming flow events are related to the proper orthogonal mode coefficients through a system of transfer functions. Components of the wakeROM that represent paths for future development are examined with the aim of generalizing the wakeROM for control of existing wind plants and design of future resources.

Data-Driven reduced-order model

A modal basis is defined from three-dimensional fields of turbulent flow data through the POD [1], which at its core seeks to describe the kernel of the decomposition (the correlation tensor) as a series of spatially-coherent POD modes ($\Phi^{(i)}$) and their respective time-varying coefficients ($a_i(t)$). Mode interaction is quantified though a series of parameters that relate the coefficients in increasing order [2]. The POD basis is truncated to $N=32$ modes (44.5% of the TKE), and coefficients are combined as,

$$\mathrm{d}a_i/dt = D_i + \sum_{j=1}^{N}[L_{ij}a_j] + \sum_{j,k=1}^{N}[Q_{ijk}a_ja_k] + \sum_{j,k.l=1}^{N}[C_{ijkl}a_ja_ka_l]$$

where parameters D_i, L_{ij}, Q_{ijk}, and C_{ijkl}, imply constant, linear, quadratic, and cubic mode interaction.

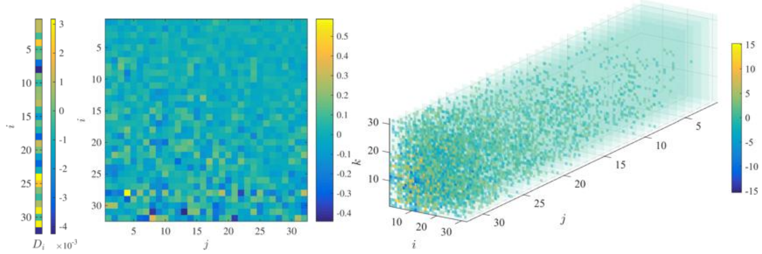

***Figure** 1: Parameters denoting constant (left), linear (center), quadratic (right) mode interaction.*

The polynomial combinations of the coefficients are reorganized as a vector $\boldsymbol{A}$, the unknown parameters as $\boldsymbol{X}_i$, and the time derivatives of a_i as $\boldsymbol{B}$. Thus, the solution of dynamical system is more easily determined as a minimization of the numerical error of $\chi^2 = ||\boldsymbol{AX} - \boldsymbol{B}||^2$. Tikhonov regularization is employed by modifying the minimization above to fit the form $\chi^2 = ||\boldsymbol{AX} - \boldsymbol{B}||^2 - \rho\, ||L(\boldsymbol{X} - \boldsymbol{X}_0)||^2$, offering an additional constraint on the vector of parameters being sought. The regularization field is denoted as ρ, a discrete approximation matrix of a differential operator is introduced as L, and an initial estimate of vector of parameters is considered as $\boldsymbol{X}_0$. Interested readers are referred to the regularization tools' documentation [3] for more details.

Generalizing the wakeROM

The wakeROM algorithm is mapped in Figure 2. Current results arise from a single option for each process, and limited input data. Colored blocks that indicate areas that merit further research. To be a generally applicable tool for design and control, the resultant model must reflect the full range of operating conditions that a wind turbine is designed to encounter. When the statistical record reflects the actual operation conditions and known wind turbine wake dynamics, the other blocks may be tuned to stabilize the wakeROM and reduce computational loads.

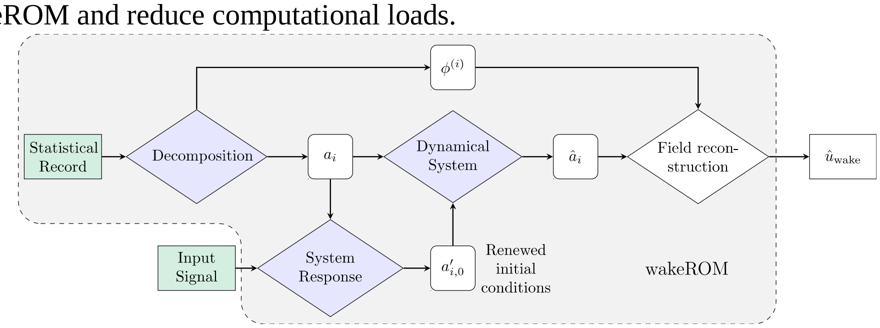

***Figure** 2: Workflow denoting key processes and data for the wakeROM.*

Low-dimensional models like the one proposed here represent a possible path for computationally efficient wake modeling reflective of incoming flow events, leading to real-time control strategies for wind turbine wakes and optimized wind plant power production. Connecting instances of the wakeROM together as suggested in Figure 3 offers a direct means of providing feedback to the network. Wind turbine operating parameters for yaw angle of the rotor (θ), the blade pitch (β), and nacelle loading (Ω) are intuitive choices to tune the behavior of a wind plant.

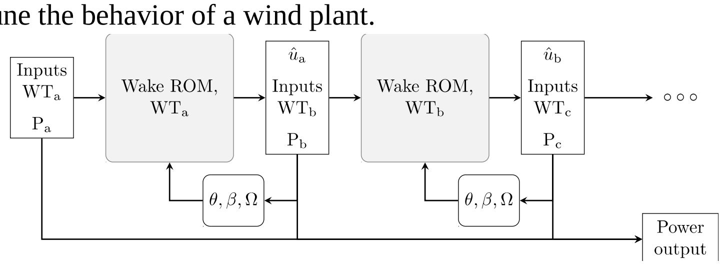

***Figure** 3: Networked instances of a generalized wakeROM for modeling, control, and optimization.*

Efficient numerical models of wind turbine wakes are necessary for optimal design and operation of future wind plants. The computational cost of LES prohibits the method from being applicable for continuous-time modeling or monitoring of wake flows. Modeling and prediction as undertaken here present a possible path forward for diagnosing control schemes and monitoring techniques easily and uniquely adapted to wind turbines in the field.

References

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